Description of Command and Control Networks in Coq

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**Abstract.** A command and control (C2) system can be defined as any group of individuals organized hierarchically in which higher-ranking individuals can issue directions to their subordinates with a certain goal in mind. We present a model for representation of command and control networks in Coq, utilizing the proof assistant’s implementation of data structures, including examples of functions and properties that may be relevant in a C2 system.

**Keywords:** Coq, proof assistant, command and control, tree data structure

1. Introduction

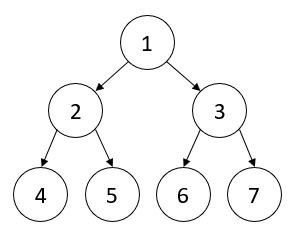
A command and control network is any system in which individuals or entities which possess authority over other individual may apply that authority with the aim of achieving a certain objection. While the term can be used in various contexts, it is commonly used in reference to a military system. **[source?]** \*\*\*

Apresentamos aqui uma definição de uma rede de controle em Coq. \*\*\*

Para demonstrar como aplicar os axiomas que definimos, vamos definir primeiro definir exemplos de redes. Estas redes consistem de três tipos de objetos: nós, locais (cada local sendo associado a um único nó), e áreas (que podem conter vários nós ou locais).

1. Representation of a Network in Coq
   1. Basic Concepts

The nodes represent the individuals in a C2 network. The hierarchy between these individuals is represented by a connected and acyclic graph, i.e., a tree. The root of the tree represents the leader in our C2 system, with each edge indicating which individuals are direct subordinates of which. By definition, we have established that each individual has only one direct superior. Figure 1.2 shows an example of a graph representing such a system, with individual 1 as the leader, 2 and 3 as its direct subordinates, and so on.



**Fig. 1.** Directed graph representing the hierarchy in a command and control network.

* 1. Defining a Network

Now, let us describe how to represent these C2 graphs in Coq. To do so, we need to define a data structure. Coq provides us with the means to do so by the Structure operator.

Require Import Coq.Lists.List.

Section nets.

Structure net : Type := {

nodes : nat ;

leader : nat ;

superior : list (nat \* nat) ;

second\_in\_command : nat := get\_second superior leader ;

parent : nat -> nat := get\_parent superior ;

children : nat -> list nat := get\_children superior ;

is\_parent : nat -> nat -> Prop := is\_parent\_func superior;

is\_parent\_bool : nat -> nat -> bool := is\_parent\_func\_bool superior;

node\_order : nat -> nat := get\_node\_order ;

sorted\_superior : list (nat \* nat) := sort superior ;

node\_level : nat -> nat := get\_level superior ;

}.

Here, we have defined a command and control network as a structure with a single leader node and a set of nodes subordinate to this leader, who in turn can have their own subordinates, and so on. The objects and functions that make up this structure are:

The number of nodes in the network, defined here as **nodes**, which are represented by a single natural number. By definition, nodes in our model are numbered individually starting from 1 without skipping any number, so nodes will also always be equal to the highest node value in a particular network. A structure with a value of 10 assigned to the nodes field, for example, will have a total of nodes numbered from 1 to 10.

**leader** tells us the index of the node which is the network’s leader, equivalent to the root of the graph.

**superior** tells us which nodes are direct subordinates of which others. This field is a list of pairs of natural numbers representing our graph, which each pair being a single edge containing the index of two nodes (parent and child). We assume that the numbers contained within these pairs are consistent with the node values defined by nodes.

**second-in-command** is a function which tells us which node in the network is the second-in-command of the current leader and the one that should replace the current leader if necessary. It is defined as the first subordinate of the leader node, as we will describe in more detail ahead.

**parent** and **children** are functions that receive a single node (natural number) as an argument and, respectively, return the index of the superior/parent node or a list of indices indicating the children/subordinates of the node.

Note that all we have defined so far are the headers of the functions in our structure, which tell us what types they receive as arguments and which types they should return. For example, look at the definition of parent here:

parent : nat -> nat := get\_parent superior ;

We are informing Coq after the : operator that the function receives a single natural number value and also returns a natural number value. After the := operator, we tell Coq how the computation of the return value is to be done—in this case, by calling a function named get\_parent which we will define later outside of our data structure. Since this function will require the values stored in superior, the list of edges, we give that as an argument here as well.

Now, let us get into the actual implementation of these individual functions.

The **second-in-command** function, as stated, tells us which node is considered the highest-ranking subordinate of the current leader and the one that should be made the leader if the current one needs to be replaced. \*\*\*

Fixpoint get\_second (edges : list (nat \* nat)) (leader : nat) : nat :=

match edges with

| (a,b) :: edges' => if (Nat.eqb a leader)

then b

else get\_second edges' leader

| nil => 0

end.

The **get\_parent** function receives one node and needs to tell us its parent node. To do this, it recursively searches through the list of edges, comparing the second number in each one (the child node, or b) with the given value until it finds a match. When that happens, the first value of the pair (the parent node, or a) is returned. Since our model already assumes that the network is defined with each node having only one parent, there is no need to search through the rest of the list after a match is found.

Should the function finish searching the list without finding an edge whose target node matches the given value, it returns 0 by default, indicating that the node has no parent. This should happen only when the value given is the leader, i.e., the root node.

Fixpoint get\_parent (edges : list (nat \* nat)) (node : nat) : nat :=

match edges with

| (a,b) :: edges' => if (Nat.eqb node b) then a

else get\_parent edges' node

| nil => 0

end.

The **get\_children** function operates similarly to get\_parent. However, since a node can have any number of children, this function needs to return a list of natural numbers. Once again, the function recursively calls itself to Search through the list of edges, this time comparing the given value with the parent node, a, in each edge. Once a match is found, we append the child value b to the list that will be our final product and continue searching via recursion, as you can see below. At the end of the run, we will have searched through every edge and have the complete list of the node’s children.

If the node has no children, an empty list value nil will be returned.

Fixpoint get\_children (edges : list (nat \* nat)) (node : nat) : list nat :=

match edges with

| (a,b) :: edges' => if (Nat.eqb node a) then b :: get\_children edges' node

else get\_children edges' node

| nil => nil

end.

We have also defined functions that tell us if two given nodes are parent and child to each other. This question of “if” is answered in Coq by two different types, proposition (Prop) or boolean (bool).

Note the importance of capitalization here. In Coq, **False** and **True** with capital letters are interpreted as values of type **Prop**, while **false** and **true** are interpreted as values of type **bool**.

Fixpoint is\_parent\_func (edges : list (nat \* nat)) (a b : nat) : Prop :=

match edges with

| nil => False

| h :: t => h = (a,b) \/ is\_parent\_func t a b

end.

Fixpoint is\_parent\_func\_bool (edges : list (nat \* nat)) (a b : nat) : bool :=

match edges with

| nil => false

| h :: t => ((Nat.eqb (fst h) a) && (Nat.eqb (snd h) b)) || is\_parent\_func\_bool t a b

end.

Next, let us look at the **get\_level** function. This function tells us the level of a node in the hierarchy. The leader’s level is by definition 1, while the level of its direct subordinates is 2, and so on.

Fixpoint get\_level\_run\_once (edges : list (nat \* nat)) (node\_and\_depth : nat \* nat) : nat \* nat :=

match edges with

| (a,b) :: edges' => if (Nat.eqb b (fst node\_and\_depth)) then get\_level\_run\_once edges' (a, (snd node\_and\_depth) + 1)

else get\_level\_run\_once edges' ((fst node\_and\_depth), (snd node\_and\_depth))

| nil => ((fst node\_and\_depth), (snd node\_and\_depth))

end.

Definition get\_level\_run\_once\_result (edges : list (nat \* nat)) (node : nat) : nat \* nat :=

get\_level\_run\_once edges (node, 1).

Fixpoint get\_level\_run\_all (edges : list (nat \* nat)) (times : nat) (node\_and\_depth : nat \* nat) : nat \* nat :=

match times with

| 0 => ((fst node\_and\_depth), (snd node\_and\_depth))

| S n => get\_level\_run\_all edges n (fst (get\_level\_run\_once edges node\_and\_depth),

snd (get\_level\_run\_once edges node\_and\_depth))

end.

Definition get\_level (edges : list (nat \* nat)) (node : nat) : nat :=

snd (get\_level\_run\_all edges (length edges) (node, 1)).

So far, we have only given an abstract description of what our networks in Coq are like. In the next section, we will see an example of get\_level and other functions applied to actual network instances with defined elements.

* 1. Defining a Network Instance
  2. Defining Properties

Now, we can move on to using the appropriate tools in Coq to define properties that a network and its elements must have. To start with a simple example, we will define the property “in any network, the leader must be one of its elements”.

As mentioned before, the list of nodes in a network is represented by a single natural number telling us how many nodes there are, with the assumption that they are all individually numbered from 1 to the stated value. Therefore, a value of 10 in this field, for example, tells us that we have a network with 10 nodes numbered 1 to 10. Thus, in order to define that the leader is always a valid node, all we need to do is inform Coq that its index is contained in that interval.

In other words, we want to tell Coq that:

This is done in Coq fairly simply:

Definition leader\_is\_in\_net := forall n : net, leader n <= nodes n.

Outra propriedade que devemos definir afirma que nenhum nó pode ser seu próprio superior. Para isto, utilizamos o elemento superior, que é uma lista de pares de números naturais correspondes às arestas de um grafo que indica quais nós da rede possuem uma relação de superioridade direta com quais outros nós.

Definition no\_self\_superior :=

forall (n : net) (i : nat), fst (nth i (superior n) (0,0)) <> snd (nth i (superior n) (0,0)).

superior n é a função superior aplicada a uma rede n qualquer, que retorna a lista de arestas.

nth i (superior n) (0,0) retorna o i-ésimo elemento desta lista, ou seja, uma única aresta. O terceiro argumento desta função, definido aqui como o par (0,0), é um valor de retorno “default” que indicamos que a função nth deve retornar caso o valor de i informado seja inválido para a lista em questão (ou se a lista estiver vazia?)

fst e snd são funções que retornam o primeiro e o segundo elementos de um par, respectivamente, e o operador <> indica que dois valores são necessariamente diferentes. Portanto, o que estamos definindo aqui é simplesmente que a lista de arestas superior n não pode possuir um elemento (a,b) em que a e b sejam iguais.

Aplicando estes mesmos princípios, podemos definir outras especificações sobre os elementos de uma rede.

Definition leader\_is\_top := forall (n : net),

~ exists i : nat, snd (nth i (superior n) (0,0)) = leader n.

(\*Definition is\_superior\_to (n : net) (a b : nat)\*)

Podemos também definir funções que possam ser aplicadas a uma rede e seus elementos. Veja como definimos uma função que informa o número de subordinados de um nó:

Fixpoint num\_children (edges : list (nat \* nat)) (node count : nat) : nat :=

match edges with

| nil => count

| (a,b) :: edges' => if (Nat.eqb a node)

then num\_children edges' node (count+1)

else num\_children edges' node count

end.

References

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